12. *Step Functions—The Postage Stamp Problem:* Figure 1-6p shows the graph of the **greatest integer function,** $f(x) = \lfloor x \rfloor$. In this function, $\lfloor x \rfloor$ is the greatest integer less than or equal to x. For instance, $\lfloor 3.9 \rfloor = 3$, $\lfloor 5 \rfloor = 5$, and $\lfloor -2.1 \rfloor = -3$.

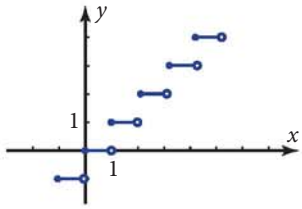
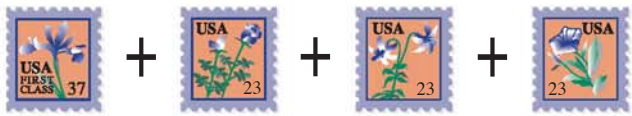


Figure 1-6p

a. Plot the greatest integer function using dot style so that points will not be connected. Most graphers use the symbol int(x) for $\lfloor x \rfloor$. Trace to $x = 2.9$, $x = 3$, and $x = 3.1$. What do you find for the three y-values?

b. In the year 2005, the postage for a first-class letter was 37 cents for weights up to 1 oz and 23 cents more for each additional ounce or fraction of an ounce. Sketch the graph of this function.



c. Using a transformation of the greatest integer function, write an equation for the 2005 postage as a function of the weight. Plot it on your grapher. Does the graph agree with the one you sketched in part b?

d. In 2005, first-class postage rates applied only until the letter reached the weight at which the postage would exceed \$3.13. What is the domain of the function in part c?

e. Check the Internet or another source to find the first-class postage rates *this* year. What differences do you find from the 2005 rates? Cite the source you used.

13. *Piecewise Functions—Weight Above and Below Earth's Surface Problem:* When you are above the surface of Earth, your weight is inversely proportional to the square of your distance from the center of Earth. This is because the farther away you are, the weaker the gravitational force between Earth and you. When you are below the surface of Earth, your weight is directly proportional to your distance from the center. At the center you would be "weightless" because Earth's gravity would pull you equally in all directions.

Figure 1-6q shows the graph of the weight function for a 150-lb person. The radius of Earth is about 4000 mi. The weight is called a *piecewise function* of the distance because it is given by different equations in different "pieces" of the domain. Each piece is called a branch of the function. The equation of the function can be written

$$y = \begin{cases} ax & \text{if } 0 \le x \le 4000 \\ \dfrac{b}{x^2} & \text{if } x \ge 4000 \end{cases}$$

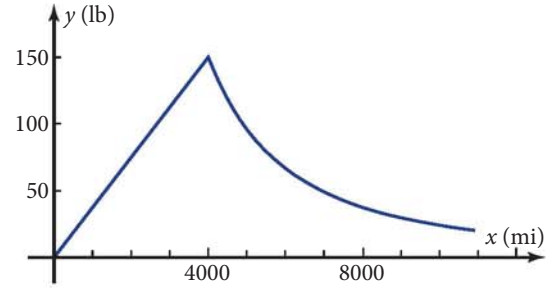


Figure 1-6q

a. Find the values of a and b that make $y = 150$ when $x = 4000$ for each branch.

b. Plot the graph of f. Use piecewise functions or Boolean variables to restrict the domain of the graph.

c. Find y if $x = 3000$ and if $x = 5000$.

d. Find the two distances from the center at which the weight would be 50 lb.

14. *Dynamic Reflection Problem:* Go to **www.flourishkh.com** and open the *Dilation* exploration. Set slider c equal to 1 and slider d equal to -1 and describe what you observe. Then set slider c equal to -1 and slider d equal to 1 and describe what you observe. Finally, set both sliders equal to -1 and describe what you observe. Explain how reflections are related to dilations.